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PLASMA, HYDRO-
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Generation of a Vortex Flow by Waves on the Surface of a Liquid

S. V. Filatov, M. Yu. Brazhnikov*, and A. A. Levchenko

Institute of Solid State Physics, Russian Academy of Sciences, Chernogolovka, Moscow region, 142432 Russia

** e-mail: makc@issp.ac.ru*

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The formation of a vortex flow in a vessel with a liquid undergoing harmonic oscillations in a vertical direction has been studied experimentally. It has been found that the vortex flow does not occur in a cylindrical vessel until the amplitude of the oscillations exceeds a threshold value, at which the Faraday parametric instability develops and azimuthal modes emerge on the surface. The vortices appear in a square vessel and in a cylindrical vessel with broken symmetry at amplitudes below the parametric instability threshold. The formation of a vortex flow is presumably caused by the interaction of surface waves propagating at an angle with respect to each other.

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INTRODUCTION

Hydrodynamics of a liquid with a free surface has been studied theoretically and experimentally for a long time. Under laboratory conditions, waves on the surface of a liquid can be excited by various means: by wave makers [1], by electrical forces acting on the interface between two liquids with different dielectric constants [2] or on the surface of a charged liquid [3], and by vertical oscillations of a vessel with a liquid as a whole [4]. In the last case, the waves first described by Faraday [5] emerge on the surface of the liquid because of developing parametric instability. As was noticed in [6], in addition to the wavy motion, a flow exhibiting chaotic behavior appears on the surface of the liquid at the parametric excitation of oscillations. As was shown later, this flow is solenoidal and, with an increase in the wave amplitude, may be sufficiently intense for the formation of a turbulent cascade [7, 8] similar to the inverse cascade in two-dimensional turbulence [9].

Despite a great number of experimental works on Faraday waves, the nature of the emergence of a flow among these waves remains unclear. Mesquita et al. [10] attempted to describe this flow as a Stocks drift [11] in a random wave field. However, the experimentally found diffusion constant of a passive scalar was an order of magnitude greater than the theoretical value.

In this work, we report the results of experiments which demonstrate that the formation of a vortex motion is associated with the two-dimensional character of a wavy motion on the surface of a liquid rather than being a specific feature of Faraday waves.

EXPERIMENTAL TECHNIQUE

A 10-mm-deep cylindrical or square vessel fixed on a vibration platform was filled with distilled water. The inner diameter of the cylindrical vessel was 65 mm, and the side of the square vessel was 50 mm. The vessel executed vertical harmonic oscillations with the amplitude S and the frequency ω_p. In the reference frame associated with the vessel, the liquid is in a time-dependent gravitational field with the acceleration equal to the sum of the acceleration of gravity g and the alternating acceleration $g\beta\cos\omega_p t$ of the vessel, where $\beta = S\omega_p^2/g$ is the dimensionless amplitude of the alternating acceleration. The acceleration was measured by an accelerometer attached to the vessel wall.

There are two mechanisms of creation of waves in such a system. One of them is associated with the presence of a meniscus on the surface of the liquid adjoining the vessel wall. The meniscus periodically changing its shape in the alternating gravitational field serves as a source of a surface wave. For the efficient excitation of a standing wave, the platform vibration frequency ω_p should be close to the resonance frequency ω_n of oscillations of the liquid surface in the vessel. In the cylindrical vessel, this mechanism excites the radial modes of oscillations of the surface of the liquid described by the Bessel function of the first kind

$$\eta(r,t) = A\cos(\omega_n t)J_0(k_n r), \qquad (1)$$

where n is the number of the resonance; k_n and ω_n are the wavenumber and frequency of the mode, respectively; and A is the amplitude of oscillations of the surface of the liquid in the center of the vessel. In the square vessel, plane waves propagating normally from each wall form a pair of standing waves

$$\eta(x,y,t) = \eta_x \cos(\omega_n t)\cos(k_n x) + \eta_y \cos(\omega_n t)\cos(k_n y). \quad (2)$$

The dispersion relation between the frequency and wavenumber involves the surface tension σ and density ρ of the liquid and the depth h of the layer [12]

$$\omega^2 = \left(gk + \frac{\sigma}{\rho}k^3\right)\tanh(kh). \quad (3)$$

Another mechanism of the excitation of waves is associated with the parametric instability of the liquid in the alternating gravitational field. For the observation of the parametric resonance, the frequency ω_p of induced oscillations of the vessel should be twice the frequency ω_n of the resonance mode. In contrast to the previous mechanism, this one is of a threshold character owing to the presence of the damping of the surface waves: amplification of the wave does not occur until the amplitude of alternating acceleration exceeds the critical value β_c. The threshold is higher if the frequency of vessel vibrations differs from the double resonance frequency.

In addition to radial modes (1), the parametric excitation of surface waves in a cylindrical vessel can also excite the azimuthal modes

$$h(r,\varphi,t) = A\cos(\omega t)J_m(k_{n,m}r)\cos(m\varphi). \quad (4)$$

The allowed wavenumbers in this case are found from the condition of the absence of a flow of the liquid through the vertical wall of the vessel, $k_{n,\,m} = \mu_n^{(m)}/R$, where n, m are integers, R is the radius of the vessel, and $\mu_n^{(m)}$ are the roots of the equation $J'_m(x) = 0$.

To visualize the flow on the surface, we added to water a powder of PA 12 polyamide particles with an average diameter of 25–30 μm or glass spheres with a diameter of 50 μm. The density of the glass spheres was slightly lower than the density of water. The surface of the liquid was illuminated by a photographic flash in a stroboscopic regime and shot with a long exposure. As a result, we obtained the images of the tracks of the probe particles.

To find the horizontal component of the velocity of the liquid flow, the surface with the probe particles was photographed at a rate of about 5.5 fps and a flash duration of 1 ms. The velocity field was determined from pair images with the use of the PIVlab code [13]. The vorticity was calculated as

$$\Omega(x,y) = \partial V_x/\partial y - \partial V_y/\partial x, \quad (5)$$

where V_x and V_y are the components of the velocity along the x and y axes, respectively.

RESULTS AND DISCUSSION

The photograph of the water surface in the cylindrical vessel decorated by the polyamide powder is shown in Fig. 1. Vibration of the platform at a frequency of 25 Hz and an amplitude below the threshold for this frequency excites the radial mode on the surface of the liquid with $n = 6$ and the wavelength $\lambda \approx$ 10 mm. The probe particles drift toward the nodes or antinodes of standing waves depending on their density and wetting [14]. Concentric circles formed by the particles collecting at the nodes of the standing wave are clearly seen in the photograph. It should be mentioned that the vortex motion is not observed in this picture.

The amplitude of oscillations of the liquid surface increases smoothly with a gradual increase in the platform vibration amplitude. Upon the achievement of a certain amplitude of alternating acceleration β, the oscillations of the liquid surface are sharply enhanced and azimuthal mode (4) with the number m on the order of 10 emerges. We regard this amplitude of alternating acceleration as the threshold value β_c. The emergence of the azimuthal mode is accompanied by the formation of a vortex motion on the surface (Fig. 2). The photograph clearly reveals the system of three concentric regions of vortices. Each region contains 12 pairs of vortices rotating in opposite directions. The vortices of the outermost region near the vessel wall are the largest. With a further increase in the pumping amplitude, large-scale flows appear, which destroy the concentric arrangement of vortices.

The transition from the cylindrical to square vessel has a drastic effect on the conditions of the formation of a vortex system. Figure 3 shows the distributions of vorticity (5) on the water surface in the square vessel under pumping at a frequency of 45.5 Hz below and above the parametric instability threshold. Below the threshold, at $\beta/\beta_c \approx 0.9$, one can see a symmetric sys-

Fig. 1. Photograph of the liquid surface under oscillations of a cylindrical vessel at a frequency of 25 Hz and an amplitude below the critical value for the excitation of the parametric resonance.

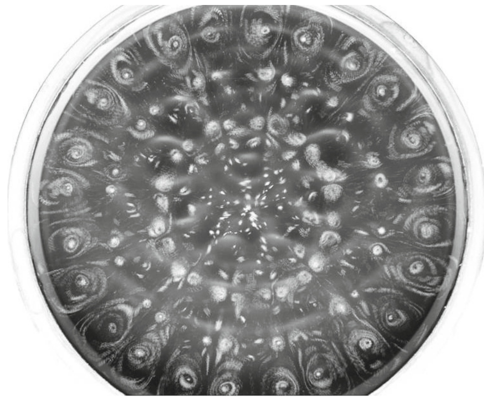

Fig. 2. Distribution of vortices over the water surface in a cylindrical vessel. The vessel vibration frequency is $\omega_p/2\pi = 45$ Hz, the amplitude of alternating acceleration is $\beta = 0.36$, and the threshold acceleration is $\beta_c = 0.26$. The azimuthal mode with $n = 4$, $m = 6$, and $\omega/2\pi = 22$ Hz is seen.

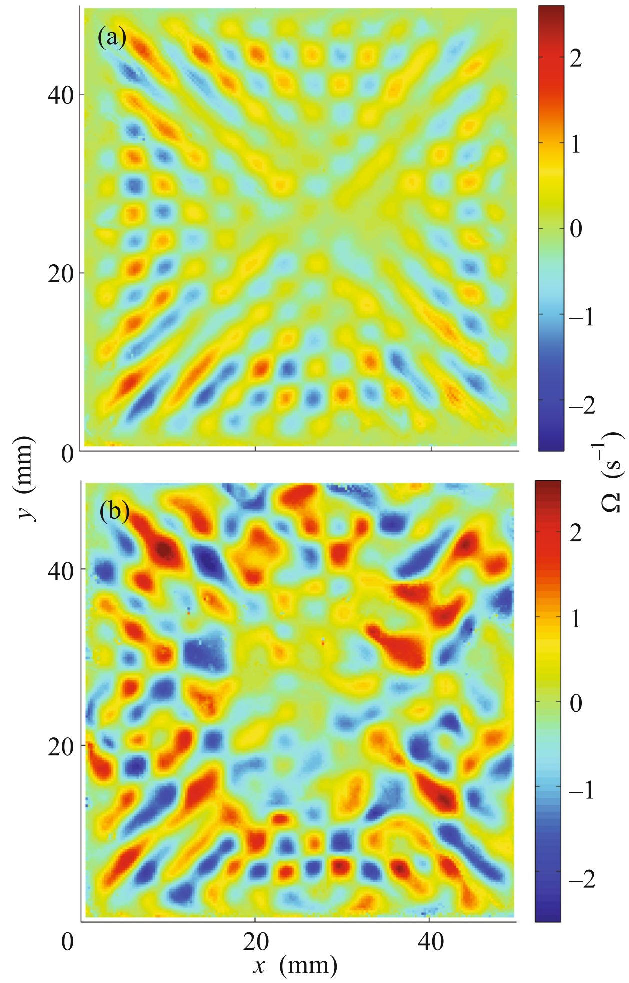

Fig. 3. (Color online) Vorticity Ω on the surface of water in a square vessel at various amplitudes of oscillations at a frequency of 45.5 Hz (a) below the threshold of the emergence of the parametric instability (the amplitude of alternating acceleration $\beta = 0.4$) and (b) after the development of the parametric instability ($\beta = 0.48$]). The threshold acceleration is $\beta_c = 0.44$.

tem of small vortices (Fig. 3a), which form a square lattice with a period equal to the wavelength of surface waves at a frequency of 45.5 Hz ($\lambda \approx 6$ mm). The symmetric structure still persists slightly above the threshold value of the acceleration, $\beta/\beta_c \approx 1.1$ (Fig. 3b). With a further increase in the pumping level, the vortices merge and become coarser owing to nonlinearity.

Figure 4 shows the Fourier transforms of the vortex structures presented in Fig. 3. At the pumping amplitude below the critical value, the surface is dominated by the structure with an inverse period of ≈ 10 cm^{-1} (Fig. 1 a) in both directions, which corresponds to the wavenumber of the mode at the pumping frequency. Figure 4b, in addition to the initial structure, reveals the structure with an inverse period of about 6 cm^{-1}, the Fourier amplitudes of which are several times higher than the Fourier amplitudes of the initial structure. An increase in the period of the vortex lattice is associated with the appearance of standing waves with the frequency $\omega_p/2$ on the water surface, the wavelength of which coincides with the period of the vortex structure that emerges above the Faraday instability threshold.

The integral vorticity $|\Omega|$ of the motion on the water surface as a function of the amplitude of alternating acceleration β in the square vessel is shown in Fig. 5a. The vorticity $|\Omega|$ increases by almost two orders of magnitude with a change in the amplitude of alternating acceleration β from 0.11 to 0.55 and, moreover, a fast increase occurs at accelerations above the parametric instability threshold. The variation of the vorticity with the acceleration amplitude β under pumping below the threshold value is described well by the power law $|\Omega| \sim \beta^{1.7}$. Since the amplitude A of standing waves excited on the surface, all other things being equal, is proportional to the amplitude of alternating acceleration β, the dependence of the integral vorticity on the wave amplitude should have the same exponent. The estimate yields $|\Omega| \sim A^2$ [15]. The difference is seemingly caused by the inhomogeneity of the vorticity field: the vorticity is higher near the edge of the vessel than in the center (Fig. 3a).

Since the vortex motion appears on the square vessel at pumping amplitudes well below the parametric instability threshold, the formation of the vortex motion in this case cannot be attributed to the specificity of the Faraday parametric instability. The fact that the structures of the vortex and wavy motions correlate with one another allows assuming that the waves directly participate in the generation of vortices. The fundamental difference of the waves in the square vessel, where the vortices are seen starting from ultimately

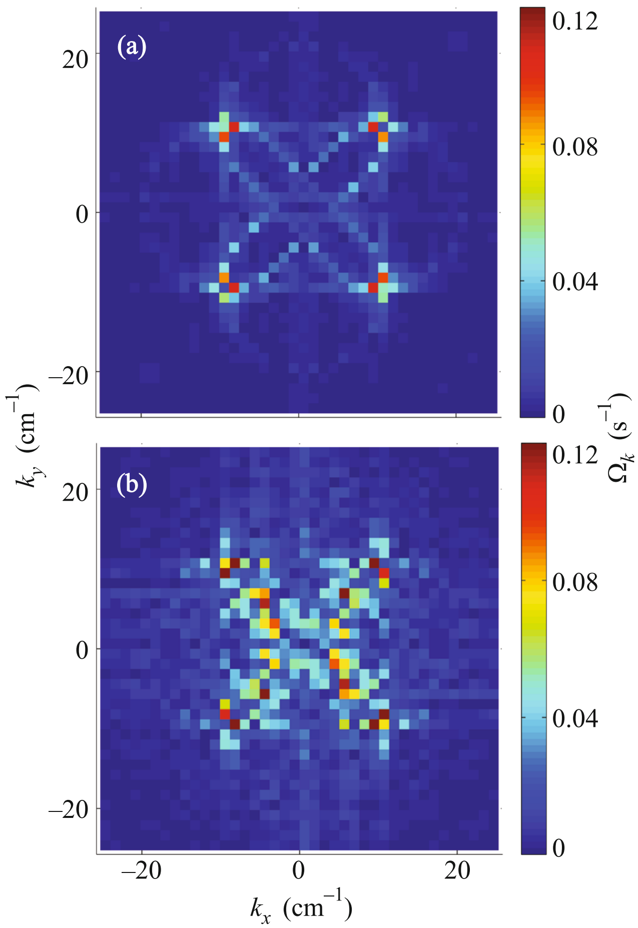

Fig. 4. (Color online) Fourier transforms of the vorticity fields shown in Fig. 3.

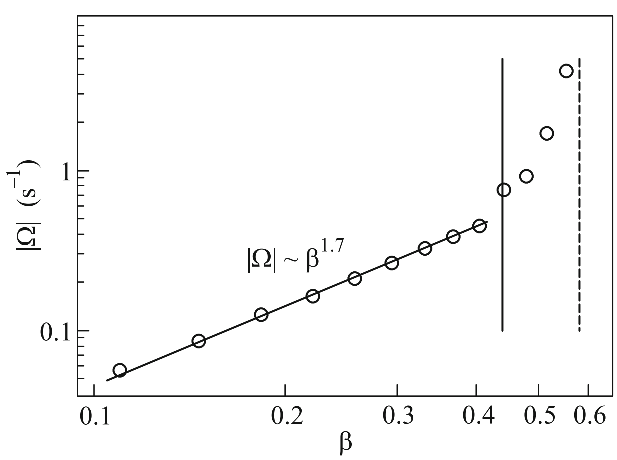

Fig. 5. Integral absolute value of the vorticity $|\Omega| = \int |\Omega(x, y)| dxdy$ on the surface of water in a square vessel versus the amplitude of alternating acceleration β. The solid vertical line corresponds to the threshold amplitude of alternating acceleration $\beta_c = 0.44$.

low pumping amplitudes, from the waves in the cylindrical vessel, where the vortices appear only upon the achievement of the instability threshold, is in the number of oscillation modes simultaneously excited on the surface of the liquid at a given frequency. In the square vessel, pair of modes (2) is always excited owing to the symmetry. In the cylindrical vessel, two different modes, radial (1) and azimuthal (4), are excited only above the parametric instability threshold. Presumably, a change in the symmetry of the cylindrical vessel that would lead to the excitation of azimuthal modes at the pumping amplitudes below the threshold value should also allow the formation of a vortex motion at the same amplitudes.

To check this assumption, the symmetry of the cylindrical vessel was broken by inserting two plastic rods with a diameter of 6.5 mm placed diametrically opposite to each other near the vessel wall. The vorticity field before and after the insertion of the rods is shown in Fig. 6. Only a radial mode is excited on the surface of the liquid in the vessel without the rods and the vortex motion does not occur. After the insertion of the rods, the azimuthal modes are well excited on the surface and a series of vortices similar to the vortex system in Fig. 2 appear along the vessel wall.

Since both modes are excited at the same frequency, their wave vectors must be close in magnitude (within the resonance width of the modes) and have different directions. In the square vessel, the angle between the wave vectors of the excited modes is 90° irrespective of the pumping frequency. In the cylindrical vessel, radial mode (1) at a large distance from the center of the vessel can be regarded as a plane wave, the wave vector of which is perpendicular to the vessel wall. The resonance mode with a low radial number *n*

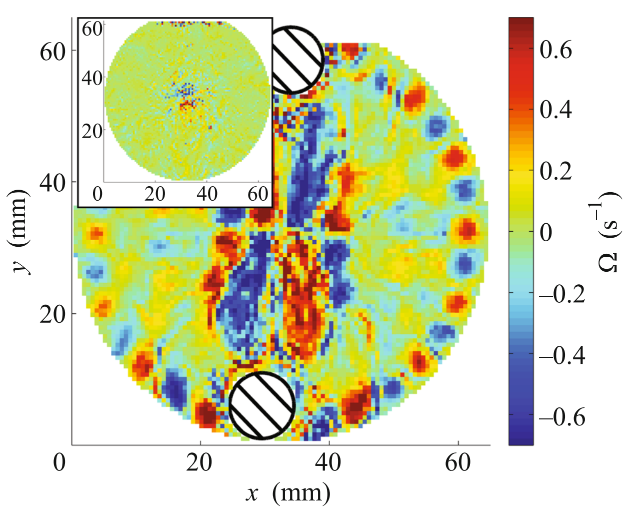

Fig. 6. (Color online) Vorticity field in a cylindrical vessel with two vertical plastic rods inside. The inset shows the vorticity before the insertion of the rods. The color scale of the vorticity is the same.

and a high azimuthal number *m*, by analogy with whispering-gallery modes of acoustic waves, can be considered as the wave propagating along the vessel boundary. For this reason, we suggest that the formation of a vortex motion in this case is due to the interaction of two surface waves, the wave vectors of which are directed at an angle with respect to one another.

CONCLUSIONS

In this work, it has been shown experimentally that standing waves on the surface of a liquid in a vessel undergoing harmonic oscillations in the vertical direction with the amplitude of alternating acceleration below the parametric instability threshold can generate a vortex flow. The structure of the vortex motion in a square vessel is a square lattice with a period equal to the wavelength of the standing waves. The vortex motion in a cylindrical vessel appears only upon the emergence of azimuthal modes, which is possible at pumping amplitudes above the parametric instability threshold. An artificial decrease in the symmetry of the cylindrical vessel, which permits the generation of azimuthal modes at low pumping amplitudes, allows the vortex motion to be formed under pumping well below the Faraday parametric instability threshold. According to these observations and taking into account the power dependence of the vorticity on the wave amplitude, it can be stated that the vortex motion appears in vessels of various symmetries when a pair of waves with noncollinear wave vectors propagate on the surface of the liquid.

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